California’s Common Core Mathematics Framework

Curriculum and Instruction Steering Committee
November 15, 2013
Sacramento, CA
CALIFORNIA DEPARTMENT OF EDUCATION

Mathematics Framework
Development Process

This chart shows the major steps of the curriculum framework development process.

All meetings are open to the public.

1. Meetings of Four Focus Groups (Educators Appointed by SSPI) Held to Solicit Input on New Framework February-March 2012


4. CFCC Meets 6 Times to Draft Framework September 2012-February 2013

5. Draft Framework Presented to IQC March 2013

6. IQC Conducts 60-Day Field Review; Draft Framework Posted on Internet April-June 2013

7. IQC’s Math Subject Matter Committee Meets; Suggests Edits to Draft Framework June-July 2013

8. IQC Meets; Recommends Draft Framework to SBE July 2013


10. IQC presents framework to SBE for Information September 2013

11. SBE Acts on Framework November 2013
Goals for the *Mathematics Framework*

- Guide the field in implementing the CA CCSSM
- Emphasize coherence across and within grade levels
- Integrate the Mathematical Practice and Content Standards
- Provide guidance on the higher mathematics course progression
What is in the Mathematics Framework?

- Introduction
- Overview of Standards Chapters
- Grade-level chapters, TK–8
- Higher mathematics chapters by course:
  - Traditional pathway
  - Integrated pathway
  - Pre-calculus, Statistics and Probability
  - Advanced Placement Probability and Statistics
  - Calculus
  - Mathematical Modeling
What is in the Mathematics Framework?

- Universal Access
- Instructional Strategies
- Supporting High-Quality Common Core Mathematics Instruction
- Technology in the Teaching of Mathematics
- Assessment
- Instructional Materials to Support the CA CCSSM (including the evaluation criteria for the mathematics adoption)
What is in the Appendix?

1. Course Placement and Sequences
2. Financial Literacy and Mathematics Education
3. Possible Adaptations for Students with Learning Difficulties in Mathematics
4. Mathematical Modeling
5. Higher Mathematics Pathways Standards Chart
What Guided the Revision of the Mathematics Framework?

- National documents and research from the Common Core State Standards Initiative
- Achieve the Core and the Progressions Documents
- The Standards for Mathematical Practice
- State Board of Education Guidelines
SBE Guidelines for the Revision of the *Mathematics Framework*

- Based on input from the focus group meetings, written comments received, and statutory requirements
- Reviewed and recommended by the IQC and approved by the SBE
- The Mathematics CFCC members develop a framework based on the guidelines
Who contributed to the draft *Mathematics Framework*?

- Focus Group members—all educators in California K–12 public schools, four regional meetings
- MCFCC members—one-half teachers, including teachers with experience teaching English learners and students with disabilities, other educators, and two content experts with Ph.Ds. in mathematics
- IQC—teachers, curriculum leaders, and administrators
- Staff of the Curriculum Frameworks and Instructional Resources Division and mathematics expert Dr. Christopher Yakes
Who contributed to the draft *Mathematics Framework*?

- The field—provided comments on the draft framework during two 60-day review periods
- County Offices of Education—held discussion forums on the 1st draft of the *Mathematics Framework*
- Common Core State Standards for Mathematics author and expert Jason Zimba
- WestEd’s California Comprehensive Center, Neal Finkelstein and Dona Meinders
- Staff of the California Department of Education’s Language Policy and Leadership Office, STEM Office, and Assessment Transition Office
Grade Four

In the years prior to grade four, students developed place value understandings, generalized written methods for addition and subtraction, and added and subtracted fluently within 1,000. They gained an understanding of single-digit multiplication and division and became fluent with such operations. Students developed an understanding of fractions as built up from unit fractions (Adapted from The Charles A. Dana Center Mathematics Common Core Toolbox 2012).

WHAT STUDENTS LEARN IN GRADE FOUR

[Note: Sidebar]

<table>
<thead>
<tr>
<th>Grade Four Critical Areas of Instruction</th>
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</thead>
</table>

In grade four instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. (CCSSO 2010, Grade 4 Introduction).

Students also work toward fluency in addition and subtraction within 1,000,000 using the standard algorithm.
Structure of the Grade Level Chapters

Ratios and Proportional Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems.
3. Use proportional relationships to solve multi-step ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

In grade six, students used ratio tables and unit rates to solve percent problems. In grade seven, students extend their work to solve multi-step ratio and percent problems (7.RP.3 ▲). They explain or show their work using a representation (e.g., numbers, words, pictures, physical objects, or equations) and verify that their answers are reasonable. Models help students identify parts of the problem and how values are related (MP.1, MP.3 and MP.4). For percentage increase and decrease, students identify the original value, determine the difference, and compare the difference in the two values to the starting value.

Examples: Multi-Step Percent Problems.

1. A sweater is marked down 30%. Its original price was $37.50. What is the price of the sweater after it is marked down?
   Solution: A simple diagram like the one shown can help students see the relationship between the original price, the amount taken off, and the sale price of the sweater. In this case, students can solve the problem either by finding 70% of $37.50, or by finding 30% of $37.50 and subtracting it.

   Seeing many examples of problems such as this one can allow students to see discount problems as taking the form \((100\% - r\%) \cdot p = d\), where \(r\) is the amount of reduction, \(p\) is the original price and \(d\) is the discounted price.

2. A shirt is on sale for 40% off. The sale price is $12. What was the original price?
   Solution: Again, a simple diagram can show the relationship between the sale price and the original price. In this case, what is known is the sale price, $12, which represents 60% of the original price. In that case, we can set up a simple equation \(0.6p = 12\), and solve for \(p\): \(p = 12 / 0.6 = 20\). The original price was $20.
Grade Level Explanations and Examples for the Standards for Mathematical Practice (Grade 3)

<table>
<thead>
<tr>
<th>Standards for Mathematical Practice</th>
<th>Explanation and Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MP.1 Make sense of problems and persevere in solving them.</strong></td>
<td>In third grade, mathematically proficient students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Students may use concrete objects, pictures, or drawings to help them conceptualize and solve problems, such as “Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase?” or “Describe another situation where there would be 5 groups of 3 or 5 × 3.” Students may check their thinking by asking themselves, “Does this make sense?” Students listen to other students’ strategies and are able to make connections between various methods for a given problem.</td>
</tr>
</tbody>
</table>
| **MP.2 Reason abstractly and quantitatively.** | Students recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. For example, students apply their understanding of the meaning of the equal sign as “the same as” to interpret an equation with an unknown. When given $4 \times ? = 40$, they might think:  
- 4 groups of some number is the same as 40  
- 4 times some number is the same as 40  
- I know that 4 groups of 10 is 40 so the unknown number is 10 |
## Grade Level Explanations and Examples for the Standards for Mathematical Practice (Grade 7)

<table>
<thead>
<tr>
<th>Standards for Mathematical Practice</th>
<th>Explanation and Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP.1 Make sense of problems and persevere in solving them.</td>
<td>In grade seven, students solve problems involving ratios and rates and discuss how they solved them. Students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “Does this make sense?” or “Can I solve the problem in a different way?” When students compare arithmetic and algebraic solutions to the same problem (7.EE.4a), they are identifying correspondences between different approaches.</td>
</tr>
<tr>
<td>MP.2 Reason abstractly and quantitatively.</td>
<td>Students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.</td>
</tr>
<tr>
<td>MP.3 Construct viable arguments and critique the reasoning of others.</td>
<td>Students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, and tables. They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. For example, as students notice when given geometric conditions determine a unique triangle, more than one triangle or no triangle (7.G.2), they have an opportunity to construct viable arguments and critique the reasoning of others. Students should be encouraged to answer questions, such as “How did you get that?”, “Why is that true?” and “Does that always work?”</td>
</tr>
</tbody>
</table>
Table Discussion

• How do the concepts explained in each of the MP1 boxes progress between the grade levels presented (3 and 7)?
• What implication do these charts have for professional learning for teachers?
• How can teachers reading the framework on their own utilize these charts?
Content of the Mathematics Framework, K–5

Focus  Coherence  Rigor

- A focus on understanding addition, subtraction, multiplication, and division (the four operations)
- Building from whole numbers in K–2 to fractions in grades 3–5
- Expectations of fluency with whole numbers and fractions
Content of the *Mathematics Framework, 6–8*

**Focus  Coherence  Rigor**

- A focus on ratio, rates, percent, and statistics and probability
- Extending operations with fractions to rational numbers
- Expectations of fluency with expressions and linear equations
Content of the *Mathematics Framework*, Higher Mathematics

**Focus  Coherence  Rigor**

- A focus on the mathematics that students need for success in college and careers
- Extending from algebraic concepts to calculus, trigonometry, and advanced statistics
- Expectation that students are college and career ready and able to utilize mathematics in their lives
Focus, Coherence, and Rigor:

As students use various counting strategies when they participate in counting activities they reinforce their understanding of the relationship between numbers and quantities and support mathematical practices such as modeling with mathematics (MP.4), the use of precise language (MP.6), and repeated reasoning to find a solution (MP.8).
Common misconceptions:

- Students sometimes treat decimals as whole numbers when making comparisons of two decimals, ignoring place value. For example, they think that 0.2 < 0.07 simply because 2 < 7.
- Students sometimes think the longer the decimal number the greater the value. For example they think that 0.03 is greater than 0.3.
Connecting to the Standards for Mathematical Practice

**Explanations and Examples**

**Task: The Comparison Game:** For this game, students have packs of 4 pairs of comparison cards, each pair corresponding to the following comparisons: heavier/lighter, taller/shorter, holds more/holds less, longer/shorter. In addition, each card pair has sample pictures on them that indicate the comparison, and furthermore, the words may be color-coded to aid students who cannot yet read the words on the cards (examples are shown in Figure 1). At the front of the room, the teacher shows the students two objects in sequence; the students must raise the appropriate card to compare the second object to the first. Several rounds are played with several different objects.

**Figure 1 Sample Cards**
Connecting to the Standards for Mathematical Practice

Explanations and Examples

Sample Problem: When Mr. Short is measured in paperclips, he is 6 paperclips tall. When he is measured in buttons, he is 4 buttons tall. Mr. Short has a daughter named Suzy Short. When Suzy Short is measured in buttons, she is 2 buttons tall. How many paperclips tall is Suzy Short?

Solution: Since Mr. Short is both 6 paperclips tall and 4 buttons tall, it must be true that 1.5 paperclips is the same height as 1 button. Therefore, since Suzy Short is 2 buttons tall, she is 2×1.5=3 paperclips tall. Also, since Suzy Short is half the number of buttons tall as her father, she must be half the number of paperclips tall.
Table Discussion

• What implication do these problem sets have for professional learning for teachers?

• How can teachers reading the framework on their own utilize these problem sets?
Content of the *Mathematics Framework*, Higher Mathematics

- Traditional Pathway (Algebra I, Geometry, Algebra II)
- Integrated Pathway (Mathematics I, II, and III)
- Precalculus
- Statistics and Probability
- Calculus
- AP Probability and Statistics
- Mathematical Modeling
Examples of Key Advances from Previous Grades or Courses

Themes from middle school algebra continue and deepen during high school. As early as grade 6, students began thinking about solving equations as a process of reasoning (6.EE.5). This perspective continues throughout Algebra I and Algebra II (A-REI).4 “Reasoned solving” plays a role in Algebra II because the equations students encounter can have extraneous solutions (A-REI.2). (excerpt from Algebra II)
## Appendix E: Higher Mathematics Pathways Standards Chart

<table>
<thead>
<tr>
<th>Number and Quantity</th>
<th>Algebra I</th>
<th>Geometry</th>
<th>Algebra II</th>
<th>Mathematics I</th>
<th>Mathematics II</th>
<th>Mathematics III</th>
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Course Sequences for Higher Mathematics

SBE Guidelines state: include a “discussion of options for middle school acceleration to support Algebra I or Integrated Mathematics I prior to ninth grade that are consistent with other Common Core states.”

Acceleration decision points at middle school—between sixth and seventh grade—and in high school, after grade eight

- Acceleration in middle school
- Doubling up, enhanced pathway, or summer bridge in high school
Course Sequences for Higher Mathematics: No Acceleration

Grade 6 -> Grade 7 -> Grade 8 -> Algebra I / Math I -> Geometry / Math II -> Algebra II / Math III -> Precalculus
Possible Course Progressions from the Standards Document

<table>
<thead>
<tr>
<th>Discipline</th>
<th>Grade Seven</th>
<th>Grade Eight</th>
<th>Grade Nine</th>
<th>Grade Ten</th>
<th>Grade Eleven</th>
<th>Grade Twelve</th>
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</thead>
<tbody>
<tr>
<td>Algebra I/Mathematics I</td>
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<td>Geometry/Mathematics II</td>
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<td>Algebra II/Mathematics III</td>
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<td>Advanced Placement</td>
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<td>Probability and Statistics</td>
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<td>Calculus</td>
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</table>
Course Sequences for Higher Mathematics: Middle School Acceleration

- Grade 6
- Grade 7 + Part of Grade 8
- Part of Grade 8 + Algebra I or Mathematics I
- Geometry or Mathematics II
- Algebra II or Mathematics III
- Precalculus
- Calculus

Acceleration Decision Point
Course Sequences for Higher Mathematics: Doubling Up

Doubling up in High School

Acceleration Decision Point

Accelerated Integrated Pathway
Course Sequences for Higher Mathematics: Enhanced & Summer Bridge

**Enhanced Pathway**

- Grade 6
- Grade 7
- Grade 8
- Enhanced Algebra I / Mathematics I
- Enhanced Geometry / Mathematics II
- Enhanced Algebra II / Mathematics III
- Calculus

**Acceleration Decision Point**

- Grade 6
- Grade 7
- Grade 8
- Algebra I / Mathematics I
- Geometry / Mathematics II
- Algebra II / Mathematics III + Pre-Calc or Summer Bridge
- Calculus

**Summer Bridge Pathway**
Algebra I Graduation Requirement

Education Code 51224.5.

(b) Commencing with the 2003–04 school year and each year thereafter, at least one course, or a combination of the two courses, in mathematics required to be completed pursuant to subparagraph (B) of paragraph (1) of subdivision (a) of Section 51225.3 by pupils while in grades 9 to 12, inclusive, prior to receiving a diploma of graduation from high school, shall meet or exceed the rigor of the content standards for Algebra I, as adopted by the State Board of Education pursuant to Section 60605.
2014 Mathematics Instructional Materials Adoption

- **Evaluation Criteria**
  - Approved by the SBE January 2013

- **Programs**
  - K–8 and Algebra I/Mathematics I
  - 35 program submissions
  - 30 recommended for adoption by the Review Panels

- **Over 100 IMRs and CREs participated**
2014 Mathematics Instructional Materials Adoption

Next Steps:

• **November 21–22, 2013:** Instructional Quality Commission acts on adoption recommendations from the Review Panels

• **January 15 –16, 2014:** State Board acts on adoption recommendations from the Instructional Quality Commission
Table Talks 1

- What do you anticipate will be the key needs of districts in your region around implementing the mathematics framework?
Table Talks 2

• How will your COE support the rollout and transition to the new framework?
Table Talks 3

• As the Mathematics Subcommittee and CDE plan regional rollouts, do you have feedback on key issues or topics that you would like to see the rollout address?
Report Out

Each table shares 3 key takeaways
Here’s How to Keep Up-to-Date on Common Core Implementation and Smarter Balanced

Subscribe to the Common Core Listserv:
join-commoncore@mlist.cde.ca.gov

Subscribe to the Smarter Balanced Listserv:
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Questions on the Mathematics Framework?

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